LESSON TITLE: Math in Basketball (by Deborah L. Ives, Ed.D)

GRADE LEVEL/COURSE: Grades 7-12 Algebra

TIME ALLOTMENT: Two 45-minute class periods

OVERVIEW

Using video segments and web interactives from ***Get the Math***, students engage in an exploration of mathematics, specifically reasoning and sense making, to solve real world problems. In this lesson, students focus on understanding the Big Ideas of Algebra: patterns, relationships, equivalence, and linearity; learn to use a variety of representations, including modeling with variables; build connections between numeric and algebraic expressions; and use what they have learned previously about number and operations, measurement, proportionality, and discrete mathematics as applications of algebra. Methodology includes guided instruction, student-partner investigations, and communication of problem-solving strategies and solutions.

In the Introductory Activity, students view a video segment in which they learn how Elton Brand, an accomplished basketball player, uses math in his work and are presented with a mathematical basketball challenge. In Learning Activity 1, students solve the challenge that Elton posed in the video, which involves using algebraic concepts and reasoning to figure out the maximum height the basketball reaches on its way into the basket by using three key variables and Elton Brand's stats.*)* As students solve the problem, they have an opportunity to use an online simulation to find a solution. Students summarize how they solved the problem, followed by a viewing of the strategies and solutions used by the ***Get the Math*** teams. In Learning Activity 2, students try to solve additional interactive basketball (projectile motion) challenges. In the Culminating Activity, students reflect upon and discuss their strategies and talk about the ways in which algebra can be applied in the world of sports and beyond.

LEARNING OBJECTIVES

Students will be able to:

- Describe scenarios that require basketball players to use mathematics and algebraic reasoning in sports.
- Identify a strategy and create a model for problem solving.
- Recognize, describe, and represent quadratic relationships using words, tables, numerical patterns, graphs, and/or equations.
- Understand the concept of a function and use function notation.
- Learn to recognize and interpret quadratic functions that arise in applications in terms of a context, such as projectile motion.
- Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.

MEDIA RESOURCES FROM THE *GET THE MATH* WEBSITE

www.getthemath.org

- The Setup **(video)** ***Optional***
  An introduction to ***Get the Math*** and the professionals and student teams featured in the program.
- Math in Basketball: Introduction **(video)**
  Elton Brand, basketball player and NBA star, describes how he got involved in sports, gives an introduction to the mathematics used in maximizing a free throw shot, and poses a related math challenge.
- Math in Basketball: Take the challenge **(web interactive)**
  In this interactive activity, users try to solve the challenge posed by Elton Brand in the introductory video segment.
- Math in Basketball: See how the teams solved the challenge **(video)**
  The teams use algebra to solve the basketball challenge in two distinct ways.
- Math in Basketball: Try other basketball challenges **(web interactive)**
  This interactive provides users additional opportunities to use key variables and players' individual statistics to solve related problems.

MATERIALS/RESOURCES

For the class:

- Computer, projection screen, and speakers (for class viewing of online/downloaded video segments)
- One copy of the "Math in Basketball: Take the challenge" answer key
- One copy of the "Math in Basketball: Try other basketball challenges" answer key

For each student:

- One copy of "Math in Basketball: Take the challenge" handoutOne copy of the "Math in Basketball: Try other basketball challenges" handout
- One graphing calculator (optional)
- Rulers, grid paper, chart paper, whiteboards/markers, overhead transparency grids, or other materials for students to display their math strategies used to solve the challenges in the Learning Activities
- Colored sticker dots and markers of two different colors (optional)
- Computers with internet access for Learning Activities 1 and 2 (optional)
  (Note: These activities can either be conducted with handouts provided in the lesson and/or by using the web interactives on the ***Get the Math*** website.)

BEFORE THE LESSON

Prior to teaching this lesson, you will need to:

- Preview all of the video segments and web interactives used in this lesson.
- Download the video clips used in the lesson to your classroom computer(s) or prepare to watch them using your classroom's internet connection.
- Bookmark all websites you plan to use in the lesson on each computer in your classroom. Using a social bookmarking tool (such as delicious, diigo, or portaportal) will allow you to organize all the links in a central location.

- Make one copy of the "Math in Basketball: Take the challenge" and "Math in Basketball: Try other basketball challenges" handouts for each student.
- Print out one copy of the "Math in Basketball: Take the challenge" and the "Math in Basketball: Try other basketball challenges" answer keys.
- Get rulers, graph paper, chart paper, grid whiteboards, overhead transparency grids, etc. for students to record their work during the learning activities.
- Get colored stickers (optional) and colored markers, for students to mark the points and construct the trajectory, or path, of the basketball in Learning Activity 1 & 2.

THE LESSON

INTRODUCTORY ACTIVITY

1. Begin with a brief discussion about sports. For instance, if any of your students play a sport, ask them to discuss the math they have used as athletes. Ask students what sports they like to watch and how they keep track of their team's progress. Ask students to discuss the mathematics that players may use to track and maximize their performance.

2. Explain that today's lesson focuses on the use of math in basketball. Ask students to brainstorm how they think mathematics might be used in the sport. (*Sample responses: knowing the rules of the game in terms of scoring, such as the shot clock timing, overtime, types and point values of shots and fouls allowed; knowing the dimensions of the court; statistical box scores, such as assists, turnovers, blocked shots, steals, field goal attempts, three-point goals and attempts, and playing time; ratios between two related statistical units, such as offensive rebounds and second-shot baskets, or two that contradict each other, such as assists and turnovers; per-minute and per-game statistics*.)

3. Explain that today's lesson features video segments and web interactives from ***Get the Math***, a program that highlights how math is used in the real world. If this is your first time using the program with this class, you may choose to play the video segment The Setup, which introduces the professionals and student teams featured in ***Get the Math***.

4. Introduce the video segment Math in Basketball: Introduction by letting students know that you will now be showing them a segment from ***Get the Math***, which features Elton Brand, an NBA basketball player. Ask students to watch for the math that he uses in his work and to write down their observations as they watch the video.

5. Play Math in Basketball: Introduction**.** After showing the segment, ask students to discuss the different ways that Elton Brand uses math in his work. *(Sample responses: He uses math to help improve his performance by using three key variables to influence his free throw shot; he uses acceleration of gravity or downward pull, the ball's initial vertical velocity, and his release height to figure*

out the height of the basketball at any given time; he uses statistics to maximize the height of the basketball so it has the best chance of going into the basket.)

6. Ask students to describe the challenge that Elton Brand posed to the teens in the video segment. *(The challenge is to use the three key variables and his stats to figure out the maximum height the ball reaches on its way into the basket in order to make a free throw shot.)*

LEARNING ACTIVITY 1

1. Explain that the students will now have an opportunity to solve the problem, which involves using the Fast Break Stats for information about the three key variables (acceleration of gravity, initial vertical velocity, release height) and Elton's stats.

2. Ask students to think of situations in their daily life where they may need to apply the concept of maximizing. *(Sample response: finding the best price to charge for the school play to get the most people to attend while still making a profit.)*

3. Discuss why you would need to maximize the height of the basketball trajectory. *(Sample responses: to make sure it reaches the hoop; the higher the shot, the further from the basket it peaks or reaches maximum height, increasing the likelihood the player will make the shot; higher arcs require a player to have more strength and use the proper mechanics.)*

4. Review the following terminology with your students:
   - **Coordinates:** *an ordered pair of numbers that identify a point on a coordinate plane.*
   - **Function:** *a relation in which every input (x-value) has a unique output (y-value).*
   - **Acceleration of Gravity:** *causes a ball to speed up, or accelerate, when falling at a rate of -32 ft/sec^2. Use only downward pull or half of -32 ft/sec^2, which is -16 t^2.*
   - **Initial Vertical Velocity:** *the angle and speed when the ball leaves the player's hand. Multiply by time to get the vertical distance traveled.*
   - **Release Height:** *the starting position of the ball when it leaves the player's hand.*
   - **Trajectory:** *the path that a basketball follows through space as a function of time.*
   - **Maximum Height:** *the value in the data set where the basketball reaches its greatest vertical distance at a given time on its way into the basket.*
   - **Parabola:** *the graph of a function in the family of functions with parent function $y = x^2$.*
     - *The path of the ball when thrown is a trajectory represented by a parabola which can be modeled mathematically with a quadratic equation. This equation represents the position of the path over time.*

- *The height (h) of a ball, in feet, at a given time (t) is represented by the equation* $\boldsymbol{h(t) = -16t^2 + v_o t + h_o}$ *where* $\boldsymbol{v_o}$ *is the initial vertical velocity and* $\boldsymbol{h_o}$ *is the initial height.*
- **Vertex:** *the highest point of the parabola.*

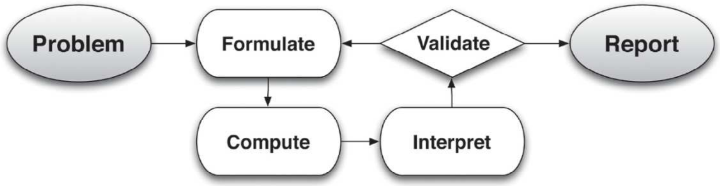
5. Distribute the "Math in Basketball: Take the challenge" handout. Let your students know that it is now their turn to solve the challenge that Elton Brand posed to the teams in the video. Explain that in the activity, students should use the Fast Break Facts for information about the three key variables and Elton's stats to figure out the maximum height the ball reaches on its way into the basket when making a free throw shot.

6. Ask students to work in pairs or small groups to complete the "Math in Basketball: Take the challenge" handout. Use the "Math in Basketball: Take the challenge" answer key as a guide to help students as they complete the activity. *Note: The handout can be used by itself or in conjunction with the "Math in Basketball: Take the challenge" activity on the website.*
   - ***If you have access to multiple computers****, ask students to work in pairs to explore the interactive and complete the handout.*
   - ***If you only have one computer****, have students work in pairs to complete the assignment using their handouts and grid or graph paper and then ask them to report their results to the group and input their solutions into the online interactive for all to see the results.*

7. Review the directions listed on the handout.

8. As students complete the challenge, encourage them to use the following 6-step mathematical modeling cycle to solve the problem:
   - *Step 1: Understand the problem:* Identify variables in the situation that represent essential features. *(For example, use the three key variables: acceleration of gravity, Elton's initial vertical velocity, and his release height.)*
   - *Step 2: Formulate a model* by creating and selecting multiple representations. *(For example, students may use visual representations in sketching a graph, algebraic representations such as combining the three key variables and Elton's stats: 24 ft/sec and a release height of 7 feet to write an equation that models the projectile motion, or an explanation/plan written in words.)*
   - *Step 3: Compute* by analyzing and performing operations on relationships to draw conclusions. *(For example, operations include calculating the values of t when the ball reaches a height of 10 feet, the value of t when the ball reaches a maximum height, and the maximum height of the basketball at this time.)*
   - *Step 4: Interpret* the results in terms of the original situation. *(The results of the first three steps should be examined in the context of the challenge to maximize the height of the basketball during the free throw shot using Elton's release height and initial vertical velocity, as well as the acceleration of gravity.*
   - *Step 5:* Ask students to *validate* their conclusions by comparing them with the situation, and then either improving the model or, if acceptable,

- *Step 6:* Report on the conclusions and the reasoning behind them. *(This step allows students to explain their strategy and justify their choices in a specific context.)*



Ongoing Assessment: Ask students to reflect upon the following:
- How can you combine the three key variables: acceleration of gravity, initial vertical velocity, and release height, to determine the maximum height of the basketball?
- At what time(s) does the ball reach 10 feet?
- At what time does the ball reach the maximum height?
- Is there only one path or trajectory for this to occur using Elton's stats? How do you know? *(You may wish to have students solve graphically to determine that this is the path using the given stats.)*

9. After students have completed the activity, ask students to share their solutions and problem-solving strategies with the class through discussion and visual materials, such as chart graph paper, grid whiteboards, overhead transparency grids, etc. Encourage students to discuss how their strategy helped (or didn't help) figure out the maximum height of the path of the ball during the free throw shot. Ask students to discuss any difficulties they faced in completing the challenge and how they overcame those obstacles.

10. As students present their solutions, ask them to discuss the mathematics they used in solving the challenge. *(Sample responses: Using a graphical model by plotting (time, distance) points for the start time and release height (0, 7), and the end time and rim height (t, 10) on a coordinate graph; representing functions using a mathematical model such as a table of values; identifying variables and writing expressions and/or a quadratic equation; using the properties of the graph of the equation to find the value of the x-coordinate of the vertex* $(-\frac{b}{2a})$*, then solving the equation for t to find the maximum height at this time; using a quadratic equation and solving by factoring, completing the square, or the quadratic formula.)*

11. Introduce the Math in Basketball: See how the teams solved the challenge video segment by letting students know that they will now be seeing how the teams in the video solved the basketball challenge. Ask students to observe what strategies the teams used and whether they are similar to or different from the strategies presented by the class.

12. Play Math in Basketball: See how the teams solved the challenge. After showing the video, ask students to discuss the strategies the teams used and to compare them to the strategies presented by the class. How are they similar? How are they different?

During the discussion, point out that the two teams in the video solved the basketball challenge in two distinct ways. Discuss the strategies listed in the "Math in Basketball: Take the challenge" answer key, as desired.

LEARNING ACTIVITY 2:

1. Go to the Math in Basketball: Try other challenges interactive. Explain to your students that they will use the web interactive to solve a series of problems similar to the one Elton Brand presented in the video segment. In this multi-level activity, students are challenged to use the 3 key variables, using a choice of player stats, to figure out the maximum height the ball reaches on its way into the basket to make the shot. Choices include: Initial Vertical Velocity of 5 feet, 6 feet, or 8 feet; Release Height of 20 ft/sec, 22 ft/sec, or 24 ft/sec. Students are encouraged to use the 3 key variables and the stats to calculate the ball's height, *h*, at a given time, *t*, by setting up an equation to get started.

   Note: As in Learning Activity 1, you can conduct this activity with one computer and an LCD projector in front of the entire class or your students can work in small groups on multiple computers. This can also be assigned to students to complete as an independent project or homework using the accompanying handout as a guide.

2. Distribute the "Math in Basketball: Try other challenges" handout. Clarify and discuss the directions.

3. Ask students to complete the handout as they explore the online challenges.
   Note: If you are using one computer, have your students work in pairs to plot points on graph or chart paper and to write the quadratic equation using the three key variables and the player's stats. Have students take turns inputting their responses into the web interactive to test their choices as they determine the time(s) the ball reaches 10 feet, the time when the ball reaches maximum height, and the maximum height at this time.

4. As in Learning Activity 1, encourage your students to use the 6-step mathematical modeling cycle as they develop a strategy to solve the challenges.

5. After students have completed the activity, lead a group discussion and encourage students to share their strategies and solutions to the challenges. Ask students to discuss how they selected the equation and graphs used, and how they calculated the values for time and height using each set of player stats.

CULMINATING ACTIVITY

1. Assess deeper understanding: Ask your students to reflect upon and write down their thoughts about the following:

- How did you determine an effective strategy for solving the challenges in this lesson? What are your conclusions and the reasoning behind them? *(Sample answer: First you could find the total flight time of the ball. Since the height of the ball is a function of the time the basketball is in the air, and the path is a trajectory or parabola, it has an axis of symmetry that passes through the vertex or highest point. Students may use this fact to make a table of values, and since it is U-shaped between the two points it is at 10 feet, students may use the symmetry to include values to the left and right of the vertex. A trace function or key in a graphing calculator, as well as a sketch of the graph, may be used to solve the problem.)*

- Compare and contrast the various algebraic and graphical representations possible for the problem. How does the approach used to solve the challenge affect the choice of representations? *(Sample answers: If you decide to graph the points and then think of the basketball as an object that is traveling on a parabolic path, or trajectory, you would use this information to find the maximum height by finding the average between the two points it is at 10 feet; if you decide to write the equation of the function by combining the three key variables: acceleration of gravity, initial vertical velocity, and release height for Elton Brand or a given player, you could use transformations to write it in Standard Form for a quadratic equation, then find the times by using the quadratic formula or completing the square as algebraic strategies.)*

- Why is it useful to represent real-life situations algebraically? *(Sample responses: Using symbols, graphs, and equations can help visualize solutions when there are situations that require using data sets or statistics to maximum performance of an athlete.)*

- What are some ways to represent, describe, and analyze patterns that occur in our world? *(Sample responses: patterns can be represented with graphs, expressions, and equations to show and understand optimization.)*

2. After students have written their reflections, lead a group discussion where students can discuss their responses. During the discussion, ask students to share their thoughts about how algebra can be applied to the world of sports. Ask students to brainstorm other real-world situations which involve the type of math and problem solving that they used in this lesson. *(Sample responses: sports-related problems might include "catching air" in snowboarding, throwing a baseball or football, hitting a golf ball, and shooting a model rocket to maximize the height of the ball or rocket; maximizing the area of a garden/farm given specific fencing options; modeling relationships between revenue and cost.)*

LEARNING STANDARDS & SAMPLE END-OF-COURSE (EOC) QUESTIONS

Sample Related End-of-Course (EOC) Questions (available for download at www.getthemath.org)
These sample questions, selected from Common Core Resources, cover the same algebraic concepts explored in this lesson.

Common Core State Standards 2010
[Note: You may also wish to view Pathways 1 and 2 for Algebra I connections in the CCSS]

Mathematical Practices

- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Use appropriate tools strategically
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning

Algebra Overview

- Seeing Structure in Expressions
  A.SSE.1a, 1b, 2 Interpret the structure of expressions.
  A.SSE.3a, 3b Write expressions in equivalent forms to solve problems.
- Arithmetic with Polynomials and Rational Functions
  A.APR.1 Perform arithmetic operations on polynomials.
  A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- Creating Equations
  A.CED.1, 2 Create equations that describe numbers or relationships.
- Reasoning with Equations and Inequalities
  A.REI.1 Understand solving equations as a process of reasoning and explain the reasoning.
  A.REI.4. Solve quadratic equations in one variable.
  A.REI.4b. Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation.
- Represent and solve equations and inequalities graphically.
  A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Functions Overview (Quadratics)

- Interpreting Functions
  F.IF. 1, 2 Understand the concept of a function and use function notation.

F.IF.4, 5, 6 Interpret functions that arise in applications in terms of a context.

- Analyzing Functions using different representations
  F.IF. 7a Graph linear and quadratic functions and show intercepts, maxima, and minima
  F.IF.8a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
  F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
- Building Functions
  F.BF.1 Build a function that models a relationship between two quantities.

Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice.

Name:________________________________ Date:__________________

Math in Basketball: Take the Challenge

Student Handout

When NBA player Elton Brand steps to the free throw line, a number of key variables can influence his shot. Your challenge is to use the 3 key variables and Elton's stats to figure out the maximum height the ball reaches on its way into the basket to make the shot.

(This activity can also be completed online. Go to www.getthemath.org, click on "The Challenges," then scroll down and click on "Math in Basketball: Take the Challenge.")

FAST BREAK FACTS: KNOW THE STATS

1. **Identify what you already know.** Look at the Fast Break Facts (following the last question in this handout) for information about the 3 key variables and Elton's stats.

- The Acceleration of Gravity: ___________
- Elton's Initial Vertical Velocity : ________
- Elton's Release Height: _______________

Combine these 3 key variables used to calculate the ball's height, *h*, at a given time, *t*, by setting up an equation to get started.

$h(t)$ = ___________________________

AT WHAT TIME(S) DOES THE BALL REACH 10 FEET?

2. **Plan it out.** What strategy will you use? Select one or more representations, such as your equation or a graph (found on the last page), to **calculate the value(s) of *t* when the ball reaches a height of 10 feet.**

3. **Solve your problem**. Show all your steps. You may use the graph on the last page or show your work in the space below.

Your solution: (Round your answer to the nearest hundredth.)

- The time(s) the ball will reach 10 feet are: ____________________

AT WHAT TIME DOES THE BALL REACH THE MAXIMUM HEIGHT?

4. **Plan it out.** What strategy will you use? Select one or more representations, such as your equation or a graph (found on the last page, to **calculate the value(s) of *t* when the ball reaches its maximum height.**

5. **Solve your problem**. Show all your steps. You may use the graph on the last page or show your work in the space below.

Your solution: (Round your answer to the nearest hundredth.)

- The time the ball will reach the maximum height is: ____________________

<u>WHAT IS THE MAXIMUM HEIGHT OF THE BASKETBALL?</u>

6. **Plan it out.** What strategy will you use? Select one or more representations, such as your equation or a graph (found on the last page), to **calculate the maximum height the ball will reach on its way to the basket.**

7. **Solve your problem.** Show all your steps. You may use the graph on the last page or show your work in the space below.

Your solution: (Round your answer to the nearest whole number.)

- The maximum height of the basketball will be at: ____________________

8. **What would you do if you had to determine the maximum height for any player's release height and initial vertical velocity stats?** If you were going to email Elton Brand to **<u>explain your strategy</u>**, what would you tell him?

FAST BREAK FACTS

THE 3 KEY VARIABLES

- **The Acceleration of Gravity –** which causes a ball to speed up, or accelerate, when falling at a rate of -32ft/sec^2. Use only downward pull or half of -32ft/sec^2, which is -16t^2.
- **Initial Upward Velocity** (v_0**)** - the angle and speed when it leaves the player's hand. Multiply by time to get the vertical distance traveled.
- **Release Height (**h_0**)** - the starting position of the ball.

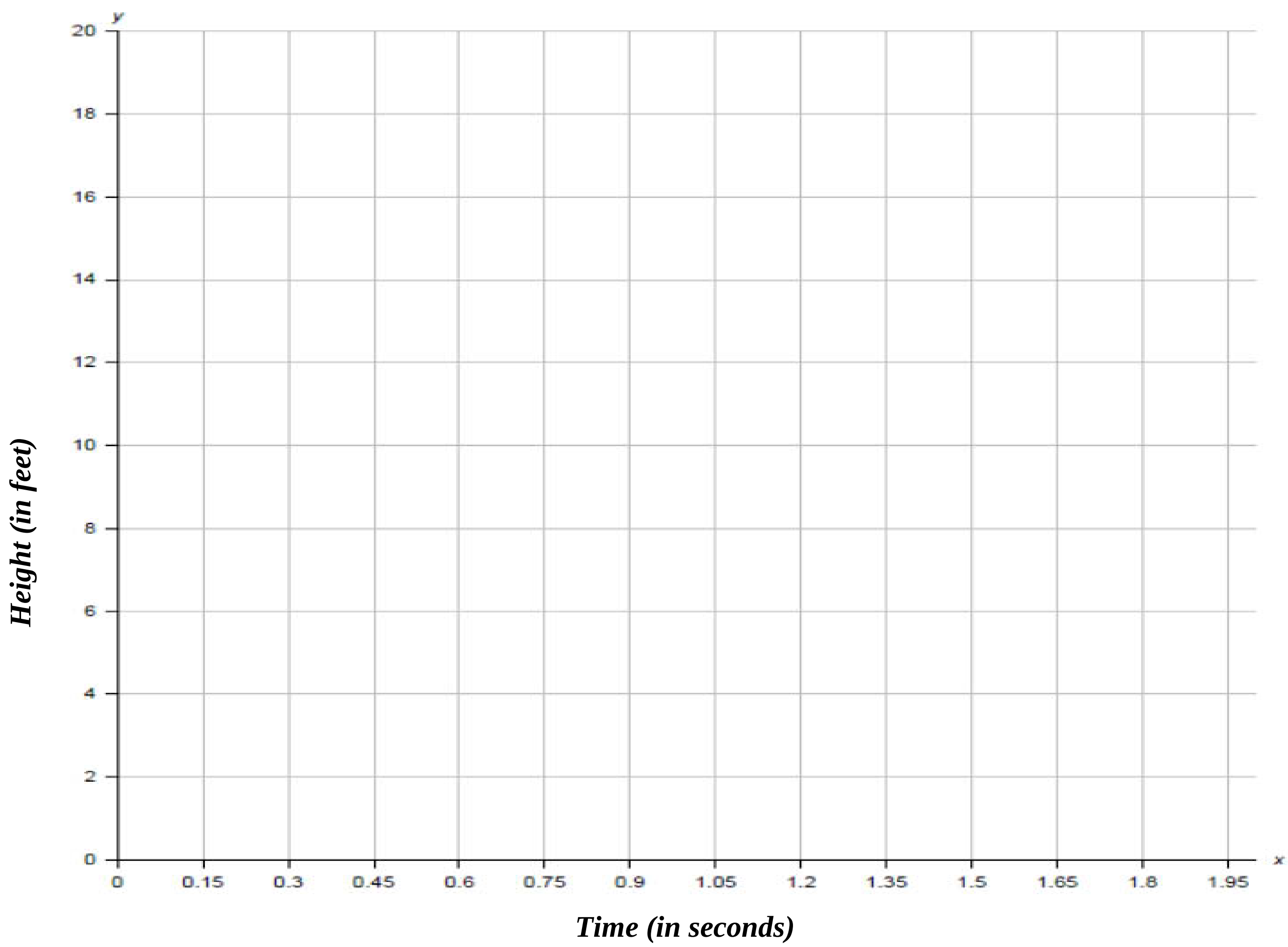
ELTON BRAND STATS

Elton's Average Initial Upward Velocity:	24 ft/sec
Elton's Average Release Height:	7.0 ft

STANDARD COURT MEASUREMENTS

Height of the basketball hoop off the floor:	10 ft
Distance from the free throw line to backboard	15 ft
Diameter of hoop/rim	18 in

GRAPH YOUR DATA



Name:_________________________________ Date:___________________

Math in Basketball: Try other basketball challenges
Student Handout

In this challenge, you get to choose a new set of player stats, then use the 3 key variables to figure out the maximum height the ball reaches during a free throw shot.

(This activity can also be completed online. Go to www.getthemath.org, click on "The Challenges," then scroll down and click on "Math in Basketball: Try other challenges.")

FAST BREAK FACTS: KNOW THE STATS

1. **Identify what you already know.** Look at the **Fast Break Facts** on the last page of this handout for information about the 3 key variables and select a player's stats from the choices below:

   - The Acceleration of Gravity: _________
   - Initial Vertical Velocity (Select one): ___ 20 ft/sec ___22 ft/sec __ 24 ft/sec
   - Release Height (Select one):____ 5 ft ____ 6 ft ____ 8 ft

   Combine these 3 key variables used to calculate the ball's height, *h*, at a given time, *t*, by setting up an equation to get started.

   h(*t*) = ______________________________

AT WHAT TIME(S) DOES THE BALL REACH 10 FEET?

2. **Plan it out.** What strategy will you use? Select one or more representations, such as your equation or a graph (found on the last page), to **calculate the value(s) of *t* when the ball reaches a height of 10 feet.**

3. **Solve your problem**. Show all your steps. You may use the graph on the last page of this handout or show your work in the space below.

Your solution: (Round your answer to the nearest hundredth.)

- The time(s) the ball will reach 10 feet are: ____________________

<u>AT WHAT TIME DOES THE BALL REACH THE MAXIMUM HEIGHT?</u>

4. **Plan it out.** What strategy will you use? Select one or more representations, such as your equation or a graph (found on the last page), to **calculate the value(s) of *t* when the ball reaches its maximum height.**

5. **Solve your problem**. Show all your steps. You may use the graph on the last page of this handout or show your work in the space below.

Your solution: (Round your answer to the nearest hundredth.)
The time the ball will reach the maximum height is: ____________________

WHAT IS THE MAXIMUM HEIGHT OF THE BASKETBALL?

6. **Plan it out.** What strategy will you use? Select one or more representations, such as your equation or a graph (found on the last page), to **calculate the maximum height the ball will reach on its way to the basket.**

7. **Solve your problem.** Show all your steps. You may use the graph on the last page of this handout or show your work in the space below.

Your solution: (Round your answer to the nearest whole hundredth.)
The maximum height of the basketball will be at: ____________________

8. **Try another player's stats.** See how the maximum height and times change when you modify the initial vertical velocity and release height. Use another copy of the handout to **s**elect a different Initial Vertical Velocity and Release Height and assist another player!

FAST BREAK FACTS

THE 3 KEY VARIABLES

- **The Acceleration of Gravity –** which causes a ball to speed up, or accelerate, when falling at a rate of -32ft/sec^2. Use only downward pull or half of -32ft/sec^2, which is $-16t^2$.
- **Initial Upward Velocity** (v_0) - the angle and speed when it leaves the player's hand. Multiply by time to get the vertical distance traveled.
- **Release Height** (h_0) - the starting position of the ball.

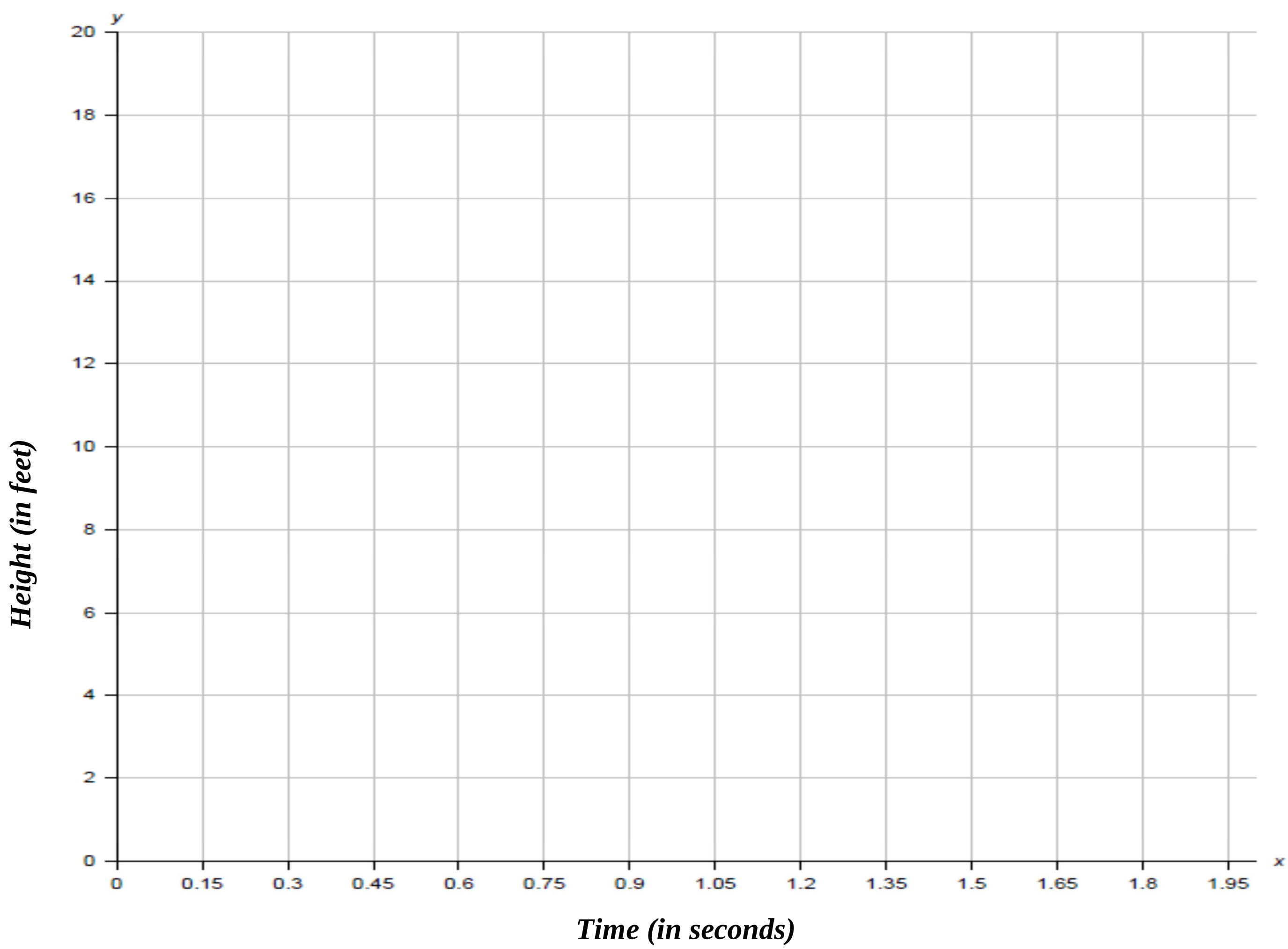
PLAYER'S STATS (Select one of each)

Initial Upward Velocity: ___ 20 ft/sec ___ 22 ft/sec ___ 24 ft/sec
Release Height: ___ 5 ft ___ 6 ft ___ 8 ft

STANDARD COURT MEASUREMENTS

Height of the basketball hoop off the floor:	10 ft
Distance from the free throw line to backboard	15 ft
Diameter of hoop/rim	18 in

GRAPH YOUR DATA



Math in Basketball: Take the Challenge

ANSWER KEY

When NBA player Elton Brand steps to the free throw line, a number of key variables can influence his shot. Your challenge is to use the 3 key variables and Elton's stats to figure out the maximum height the ball reaches on its way into the basket to make the shot.

(This activity can also be completed online. Go to www.getthemath.org, click on "The Challenges," then scroll down and click on "Math in Basketball: Take the Challenge.")

FAST BREAK FACTS: KNOW THE STATS

1. **Identify what you already know.** Look at the Fast Break Facts (following the last question in this handout) for information about the 3 key variables and Elton's stats.

- The Acceleration of Gravity: **$-16t^2$**
- Elton's Initial Vertical Velocity : **24 ft/sec**
- Elton's Release Height: ***7 feet***

Combine these 3 key variables used to calculate the ball's height, *h*, at a given time, *t*, by setting up an equation to get started.

$h(t) = -16t^2 + 24t + 7$

AT WHAT TIME(S) DOES THE BALL REACH 10 FEET?

2. **Plan it out.** What strategy will you use? Select one or more representations, such as your equation or a graph (found on the last page), to **calculate the value(s) of *t* when the ball reaches a height of 10 feet.**

Strategy A:

The height (h) of the ball, in feet, at a given time (t) is represented by the equation:

$h(t) = -16t^2 + 24t + 7$

where 24 is the initial vertical velocity and 7 is the release height

The value for t at 10 feet would occur at two points in time, one on the way up, the other at the hoop.

Substitute 10 feet for h(t): $10 = -16t^2 + 24t + 7$

Write in standard form: $0 = at^2 + bt + c$ by subtracting 10 from each side:

$0 = -16t^2 + 24t + (-3)$; where a = -16, b = 24, and c = -3

Solve algebraically using the quadratic formula, $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, or complete the square to find two values of t.

Strategy B:
Another option is to graph the equation for the height of the ball, either using a graphing calculator or paper and pencil with a table of values. Then, you can use your graph to estimate the values of *t* at which the ball reaches 10 feet.

3. **Solve your problem**. Show all your steps. You may use the graph on the last page or show your work in the space below.

Strategy A:

- Use the quadratic formula: $t = \frac{-24 \pm \sqrt{24^2 - 4(-16)(-3)}}{2(-16)} = \frac{3 \pm \sqrt{6}}{4}$

$$\approx \mathbf{0.14 \text{ or } 1.36}$$

- Complete the square:

$$\begin{aligned}
h(t) &= -16t^2 + 24t + 7 \\
10 &= -16t^2 + 24t + 7 \\
3 &= -16t^2 + 24t \\
3 &= -16(t^2 + \tfrac{3}{2}t) \\
\frac{3}{-16} &= (t^2 + \tfrac{3}{2}t) \\
\frac{3}{-16} + (-\tfrac{3}{4})^2 &= t^2 + \tfrac{3}{2}t + (\tfrac{1}{2} * (-\tfrac{3}{2}))^2 \\
\frac{6}{16} &= (t - \tfrac{3}{4})^2 \\
t &= \tfrac{3}{4} \pm \tfrac{1}{4}\sqrt{6} \approx 1.36 \text{ or } 0.14
\end{aligned}$$

Strategy B: See last page of this answer key for sample graph.

Your solution: (Round your answer to the nearest hundredth.)

- The time(s) the ball will reach 10 feet are: **0.14 and 1.36 seconds**

AT WHAT TIME DOES THE BALL REACH THE MAXIMUM HEIGHT?

4. **Plan it out.** What strategy will you use? Select one or more representations, such as your equation or a graph (found on the last page), to **calculate the value(s) of *t* when the ball reaches its maximum height.**

Strategy A:
Represented graphically, the equation for height as a function of time, or h(*t*), is a parabola. Like all parabolas, it is symmetrical, meaning that it has an axis of symmetry that passes through the vertex, or highest point. Since you now know the two values of *t* when the ball reaches a height of 10 feet, you can find the axis of symmetry by calculating the halfway point, or mean, between these two times. This will give you the value of *t* when the ball reaches its maximum height.

Strategy B:
You can use the properties of the graph of the equation for h(t) to find the value of t for the vertex or maximum point. For a parabolic function of the form **0 = at^2 + bt + c,** where a ≠ 0, the value for time *(t)* is represented by the x-coordinate of the vertex $\left(-\frac{b}{2a}\right)$ of the parabola.

5. **Solve your problem**. Show all your steps. You may use the graph on the last page or show your work in the space below.

Strategy A: Finding the mean between the two times (0.14 + 1.36) ÷ 2 = **0.75**

Strategy B: Using $\left(-\frac{b}{2a}\right) = \left(-\frac{24}{2*(-16)}\right) = \left(\frac{24}{32}\right) = \frac{3}{4}$

Your solution: (Round your answer to the nearest hundredth.)

- The time the ball will reach the maximum height is: **$\frac{3}{4}$ or .75 seconds**

WHAT IS THE MAXIMUM HEIGHT OF THE BASKETBALL?

6. **Plan it out.** What strategy will you use? Select one or more representations, such as your equation or a graph (found on the last page), to **calculate the maximum height the ball will reach on its way to the basket.**

Using the value of t, or time, when the ball reaches its maximum height, you can substitute that value into the equation you set up for h*(t)* to find the height of the ball at that time, or use graphical representation.

7. **Solve your problem.** Show all your steps. You may use the graph on the last page or show your work in the space below.

t = 0.75 seconds
h(t) = -16(0.75)^2 + 24(0.75) + 7
h(t) = 16 feet

Your solution: (Round your answer to the nearest whole number.)

- The maximum height of the basketball will be at: **16 feet**

8. **What strategy would you use if you had to determine the maximum height for another player's release height and initial vertical velocity stats?** If you were going to email Elton Brand to **explain your process**, what would you tell him?

FAST BREAK FACTS

THE 3 KEY VARIABLES

- **The Acceleration of Gravity –** which causes a ball to speed up, or accelerate, when falling at a rate of -32ft/sec^2. Use only downward pull or half of -32ft/sec^2, which is -16t^2.
- **Initial Upward Velocity** (v_0**)** - the angle and speed when it leaves the player's hand. Multiply by time to get the vertical distance traveled.
- **Release Height** (h_0**)** - the starting position of the ball.

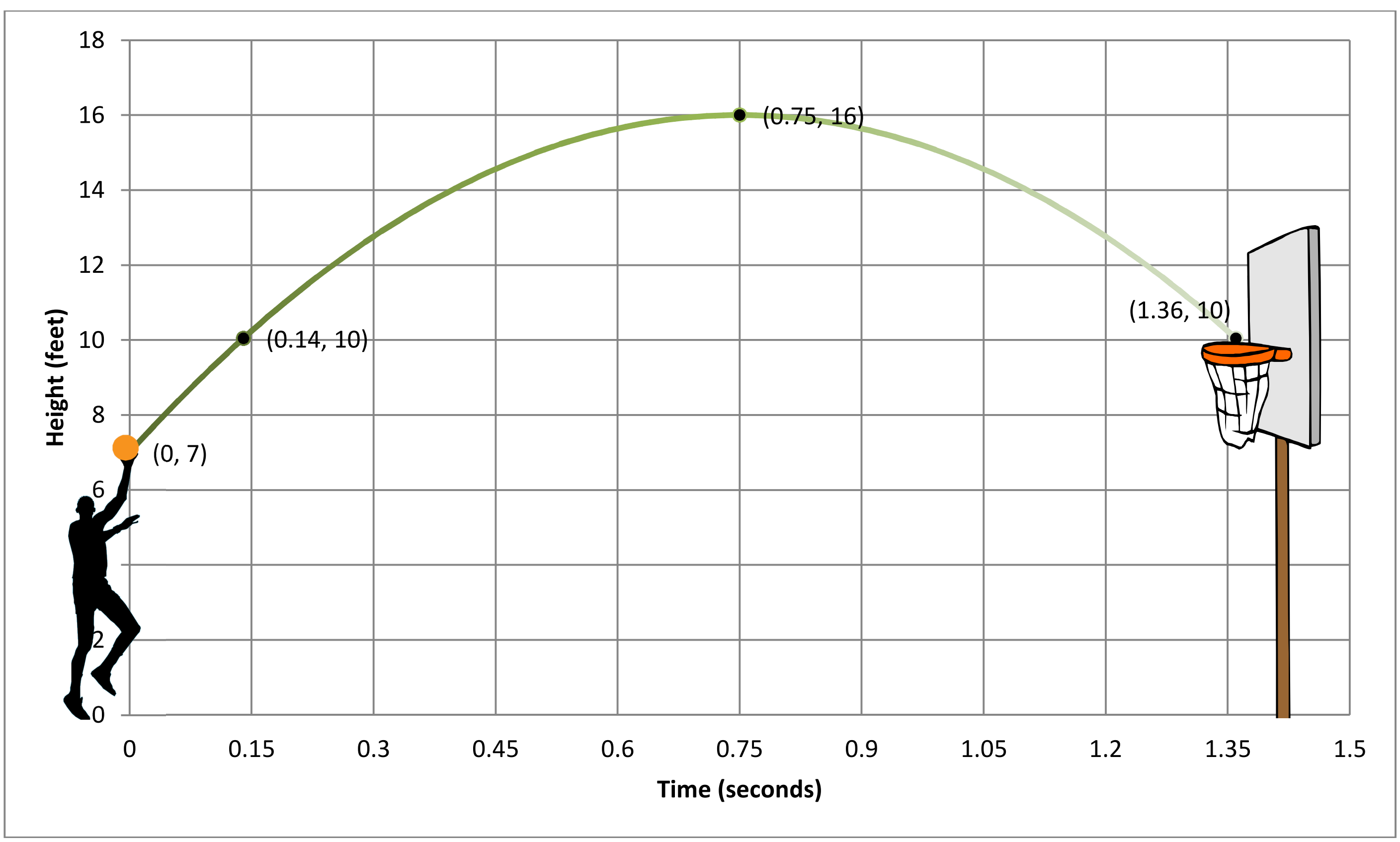
ELTON BRAND STATS

Elton's Average Initial Upward Velocity:	24 ft/sec
Elton's Average Release Height:	7.0 ft

STANDARD COURT MEASUREMENTS

Height of the basketball hoop off the floor:	10 ft
Distance from the free throw line to backboard	15 ft
Diameter of hoop/rim	18 in

GRAPH YOUR DATA



Math in Basketball: Try other basketball challenges

ANSWER KEY

In this challenge, you get to choose a new set of player stats, then use the 3 key variables to figure out the maximum height the ball reaches during a free throw shot.

(This activity can also be completed online. Go to www.getthemath.org, click on "The Challenges," then scroll down and click on "Math in Basketball: Try other challenges.")

FAST BREAK FACTS: KNOW THE STATS

1. **Identify what you already know.** Look at the **Fast Break Facts** on the last page of this handout for information about the 3 key variables and select a player's stats from the choices below:

   - The Acceleration of Gravity: ________
   - Initial Vertical Velocity (Select one): ___ 20 ft/sec ___22 ft/sec __ 24 ft/sec
   - Release Height (Select one): ____ 5 ft ____ 6 ft ____ 8 ft

   Combine these 3 key variables used to calculate the ball's height, *h*, at a given time, *t*, by setting up an equation to get started.

   h(*t*) = ______________________________

   The answer will vary depending on the stats chosen by the student, but will follow this model:
   $h(t) = -16t^2 + v_0t + h_0$ (Note: Initial Vertical Velocity = v_0; Release Height = h_0)

   For example, if the student selected Initial Vertical Velocity = 5 ft and Release Height = 20 ft/sec, the correct equation would be $h(t) = -16t^2 + 20t + 5$.

AT WHAT TIME(S) DOES THE BALL REACH 10 FEET?

2. **Plan it out.** What strategy will you use? Select one or more representations, such as your equation or a graph (found on the last page), to **calculate the value(s) of *t* when the ball reaches a height of 10 feet.**

Strategy A:

The height (h) of the ball, in feet, at a given time (t) is represented by the equation:

$$\mathbf{h(t) = -16t^2 + v_0 t + h_0}$$

[Replace initial vertical velocity and release height values based on selection above.]

The value for t at 10 feet would occur at two points in time, one on the way up, the other at the hoop.

Substitute 10 feet for h(t) and solve.

Write in standard form: $0 = at^2 + bt + c$ by subtracting 10 from each side.

Solve algebraically using the quadratic formula, $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, or complete the square to find two values of t.

Strategy B:

Another option is to graph the equation for the height of the ball, either using a graphing calculator or paper and pencil with a table of values. Then, you can use your graph to estimate the values of t at which the ball reaches 10 feet.

3. **Solve your problem**. Show all your steps. You may use the graph on the last page of this handout or show your work in the space below.
   See below for all solutions.

AT WHAT TIME(S) DOES THE BALL REACH THE MAXIMUM HEIGHT?

4. **Plan it out.** What strategy will you use? Select one or more representations, such as your equation or a graph (found on the last page), to **calculate the value(s) of *t* when the ball reaches its maximum height.**

Strategy A:

Represented graphically, the equation for height as a function of time, or h(t), is a parabola. Like all parabolas, it is symmetrical, meaning that it has an axis of symmetry that passes through the vertex, or highest point. Since you now know the two values of t when the ball reaches a height of 10 feet, you can find the axis of symmetry by calculating the halfway point, or mean, between these two times. This will give you the value of t when the ball reaches its maximum height.

Strategy B:

You can use the properties of the graph of the equation for h(t) to find the value of t for the vertex or maximum point. For a parabolic function of the form $0 = at^2 + bt + c$, where $a \neq 0$, the value for time *(t)* is represented by the x-coordinate of the vertex $\left(-\frac{b}{2a}\right)$ of the parabola.

5. **Solve your problem**. Show all your steps. You may use the graph on the last page of this handout or show your work in the space below.
   See below for all solutions.

WHAT IS THE MAXIMUM HEIGHT OF THE BASKETBALL?

6. **Plan it out.** What strategy will you use? Select one or more representations, such as your equation or a graph (found on the last page), to **calculate the maximum height the ball will reach on its way to the basket.**

   Using the value of *t*, or time, when the ball reaches its maximum height, you can substitute that value into the equation you set up for h*(t)* to find the height of the ball at that time, or use graphical representation.

7. **Solve your problem.** Show all your steps. You may use the graph on the last page of this handout or show your work in the space below.
   See below for all solutions.

ALL FINAL SOLUTIONS:
[h_0 = Release Height and v_0 = Initial Vertical Velocity]

h_0 = 5' and v_0 = 20 ft/sec

$h(t) = -16t^2 + 20t + 5$

At what times (in sec) does the ball reach 10 feet? $t = 0.35$ $t = 0.90$
At what time does the ball reach its maximum height? $t = 0.63$ (or 5/8 sec)
Maximum height = 11.25 feet

h_0 = 5' and v_0 = 22 ft/sec

$h(t) = -16t^2 + 22t + 5$

At what times (in sec) does the ball reach 10 feet? $t = 0.29$ $t = 1.09$
At what time does the ball reach its maximum height? $t = 0.69$ (or 11/16 sec)
Maximum height = 12.56 feet

h_0 = 5' and v_0 = 24 ft/sec

$h(t) = -16t^2 + 24t + 5$

At what times (in sec) does the ball reach 10 feet? $t = 0.25$ $t = 1.25$
At what time does the ball reach its maximum height? $t = 0.75$ (or ¾ sec)
Maximum height = 14 feet

---

h_0 = 6' and v_0 = 20 ft/sec

$h(t) = -16t^2 + 20t + 6$

At what times (in sec) does the ball reach 10 feet? $t = 0.25$ $t = 1.0$
At what time does the ball reach its maximum height? $t = 0.\ 63$ (or 5/8 sec)
Maximum height = 12.25 feet

h_0 = 6' and v_0 = 22 ft/sec

$h(t) = -16t^2 + 22t + 6$

At what times (in sec) does the ball reach 10 feet? $t = 0.22$ $t = 1.16$
At what time does the ball reach its maximum height? $t = 0.\ 69$ (or 11/16 sec)
Maximum height = 13.56 feet

Math in Basketball: Try other challenges *Answer Key*

h_0 = 6' and v_0 = 24 ft/sec

$h(t) = -16t^2 + 24t + 6$

At what times (in sec) does the ball reach 10 feet? $t = 0.19$ $t = 1.31$

At what time does the ball reach its maximum height? $t = 0.75$ (or ¾ sec)

Maximum height = 15 feet

---

h_0 = 8' and v_0 = 20 ft/sec

$h(t) = -16t^2 + 20t + 8$

At what times (in sec) does the ball reach 10 feet? $t = 0.11$ $t = 1.14$

At what time does the ball reach its maximum height? $t = 0.63$ (or 5/8 sec)

Maximum height = 14.25 feet

h_0 = 8' and v_0 = 22 ft/sec

$h(t) = -16t^2 + 22t + 8$

At what times (in sec) does the ball reach 10 feet? $t = 0.10$ $t = 1.28$

At what time does the ball reach its maximum height? $t = 0.69$ (or 11/16 sec)

Maximum height = 15.56 feet

h_0 = 8' and v_0 = 24 ft/sec

$h(t) = -16t^2 + 24t + 8$

At what times (in sec) does the ball reach 10 feet? $t = 0.09$ $t = 1.41$

At what time does the ball reach its maximum height? $t = 0.75$ (or ¾ sec)

Maximum height = 17 feet

FAST BREAK FACTS

THE 3 KEY VARIABLES

- **The Acceleration of Gravity –** which causes a ball to speed up, or accelerate, when falling at a rate of -32ft/sec^2. Use only downward pull or half of -32ft/sec^2, which is $-16t^2$.
- **Initial Upward Velocity (v_0)** - the angle and speed when it leaves the player's hand. Multiply by time to get the vertical distance traveled.
- **Release Height (h_0)** - the starting position of the ball.

PLAYER'S STATS (Select one of each)

Initial Upward Velocity: ___ 20 ft/sec ___ 22 ft/sec ___ 24 ft/sec

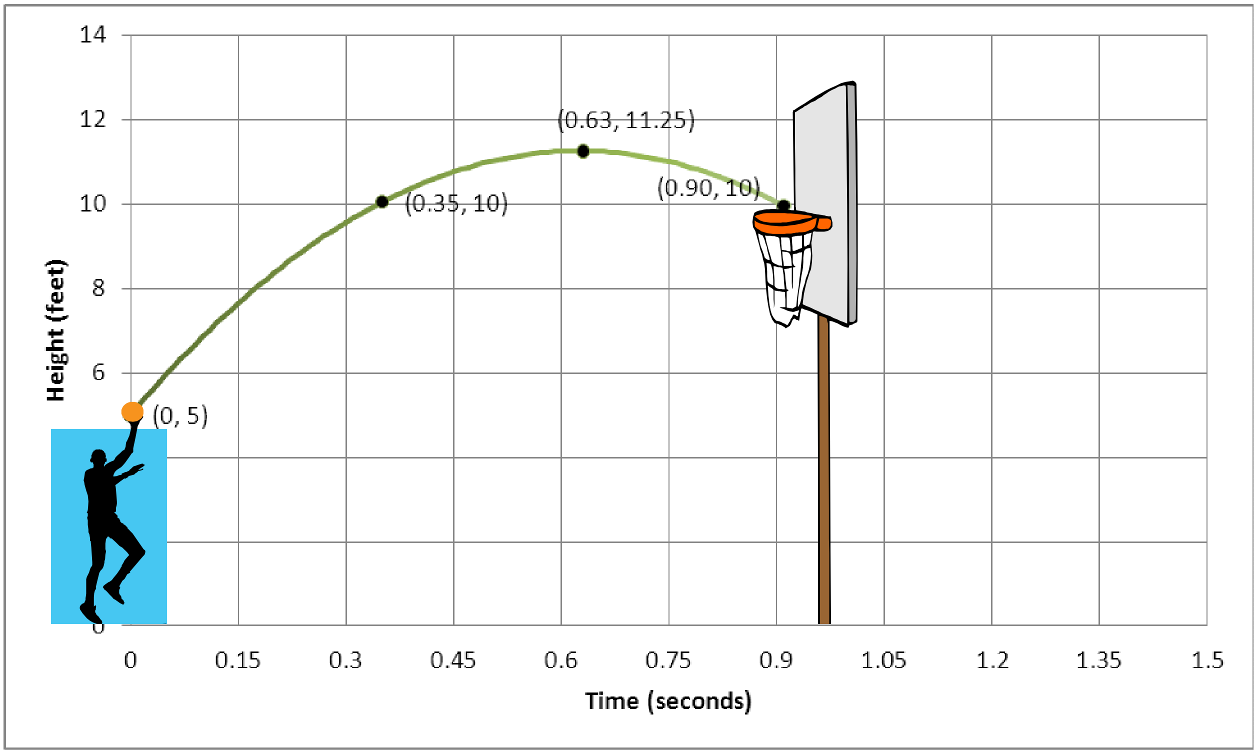
Release Height: ___ 5 ft ___ 6 ft ___ 8 ft

STANDARD COURT MEASUREMENTS

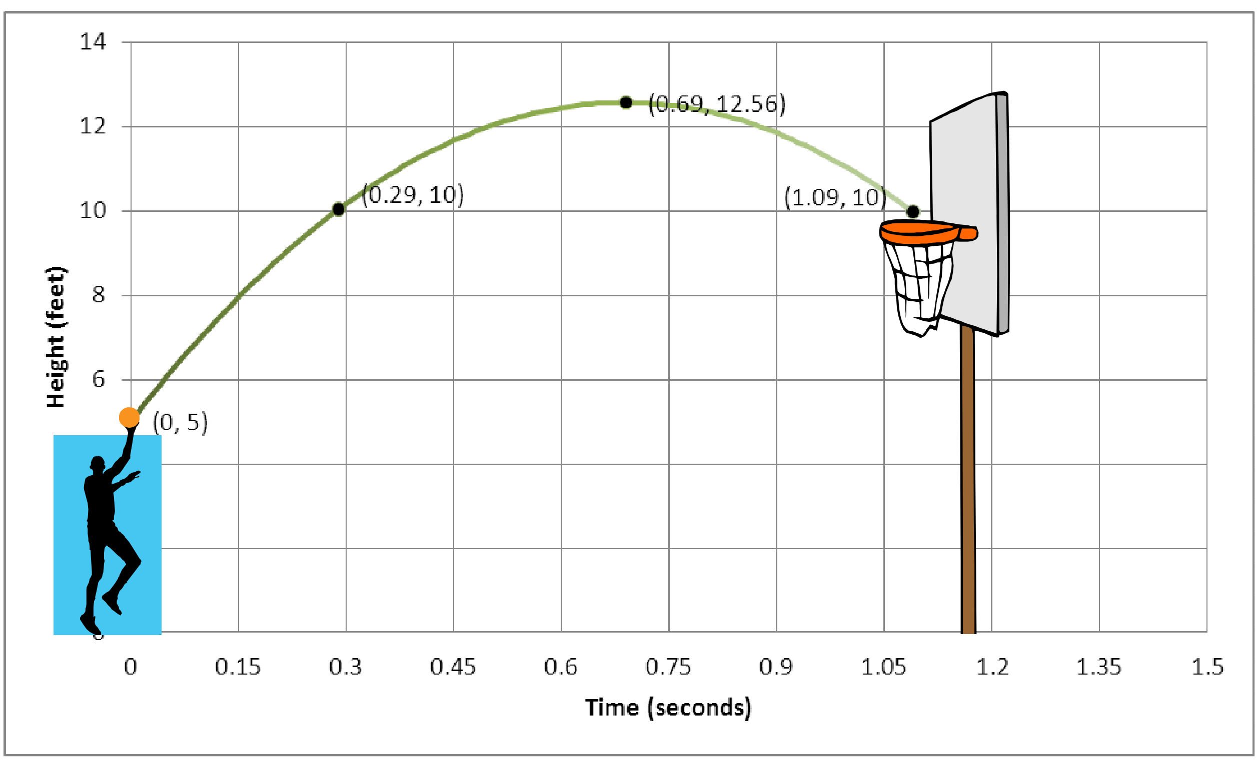
Height of the basketball hoop off the floor:	10 ft
Distance from the free throw line to backboard	15 ft
Diameter of hoop/rim	18 in

GRAPH YOUR DATA

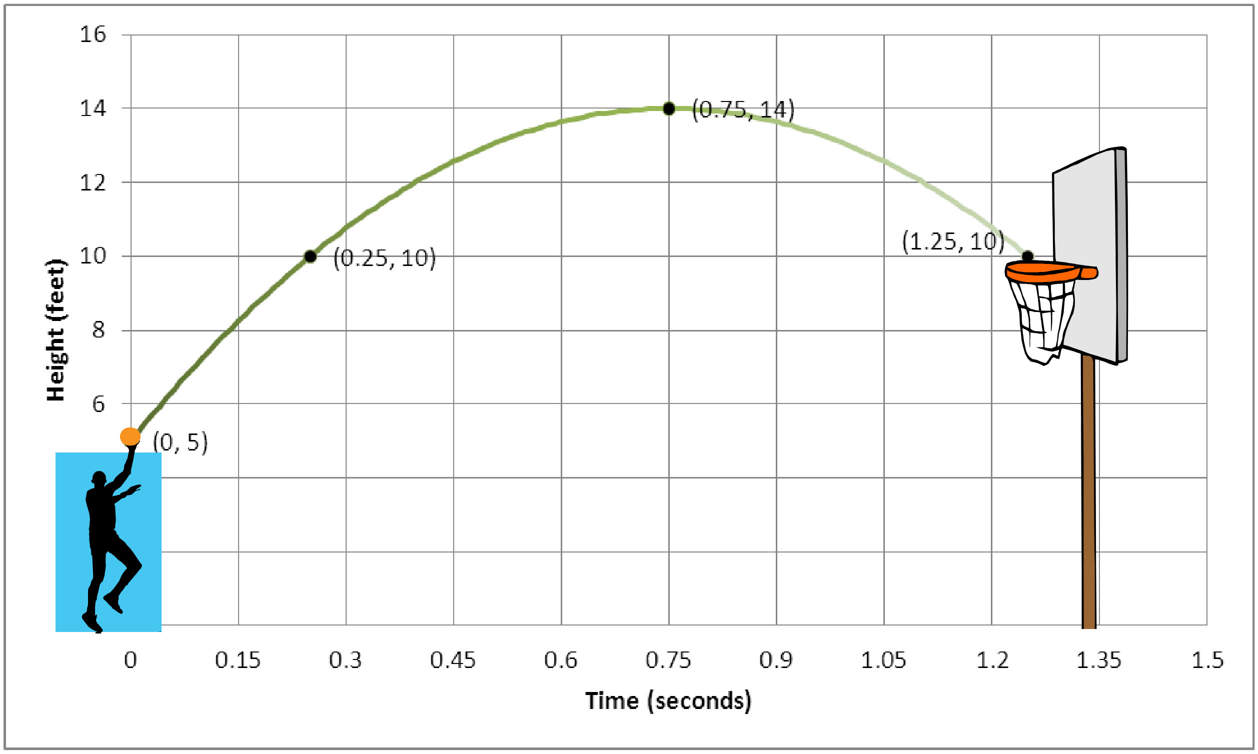
Release height = 5 ft, Initial Vertical Velocity = 20 ft/sec



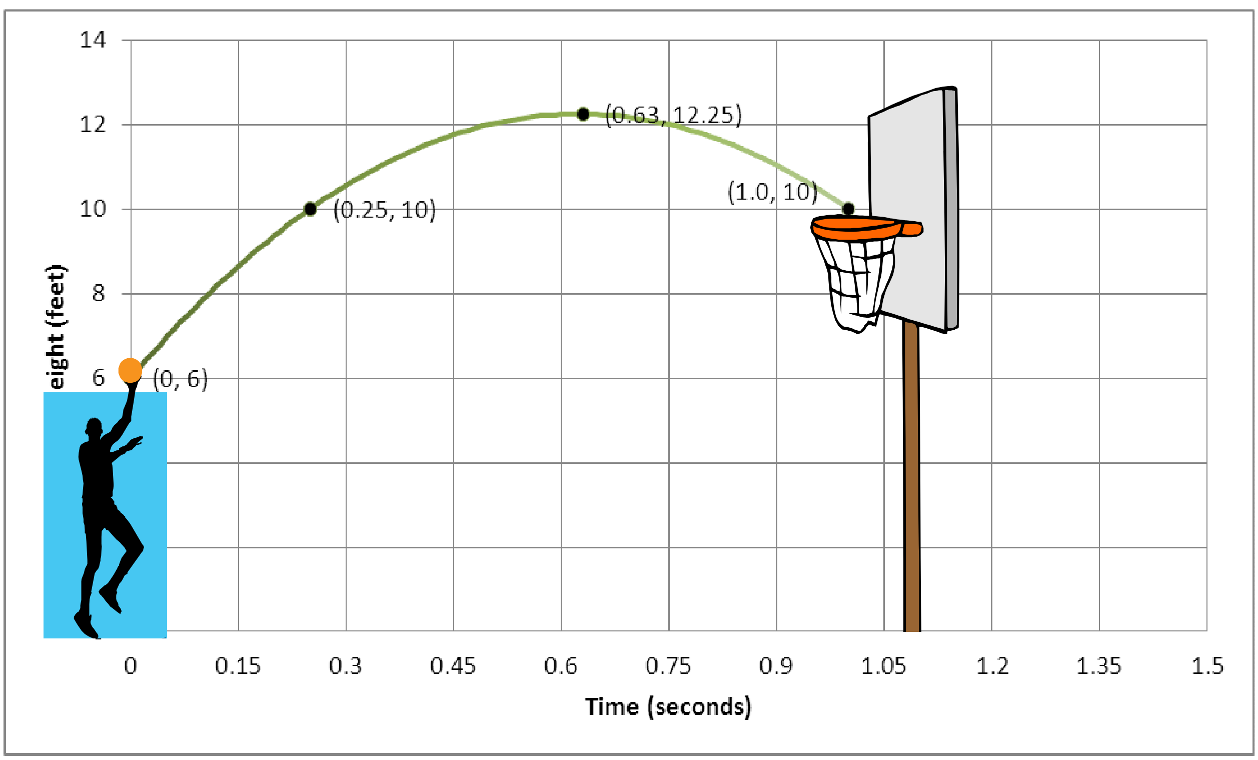
Release height = 5 ft, Initial Vertical Velocity = 22 ft/sec



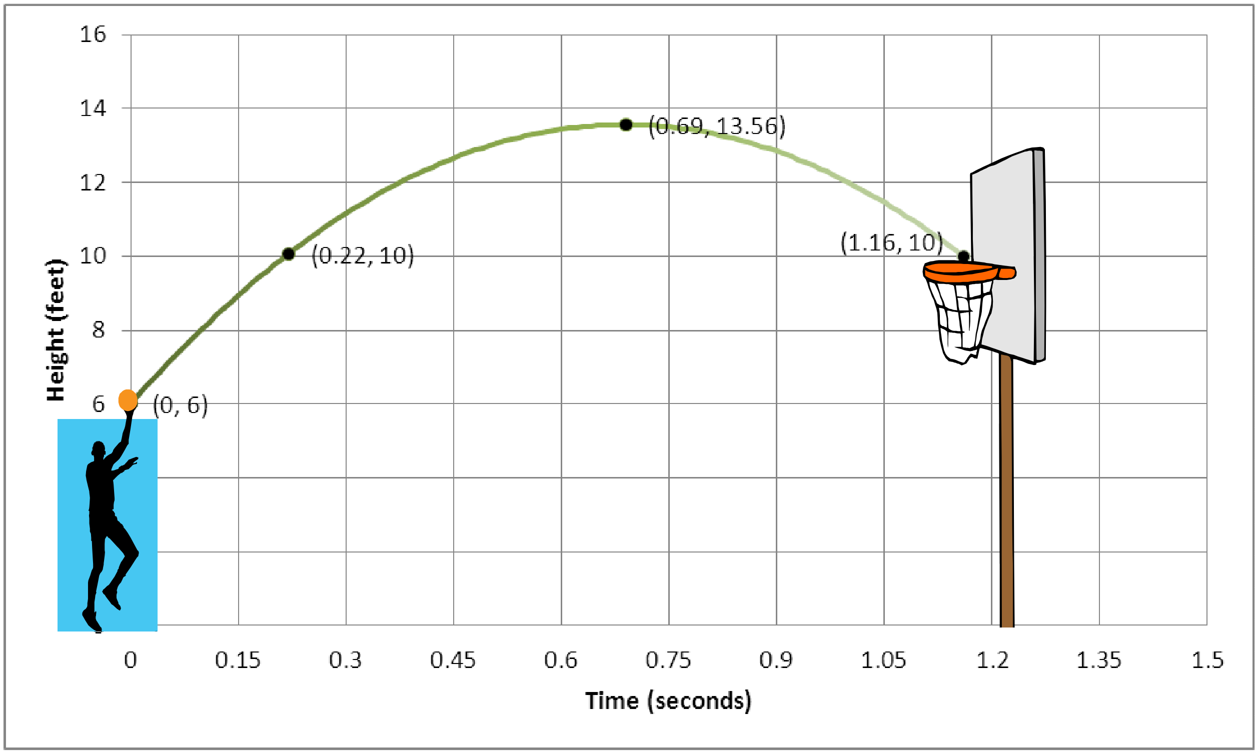
Release height = 5 ft, Initial Vertical Velocity = 24 ft/sec



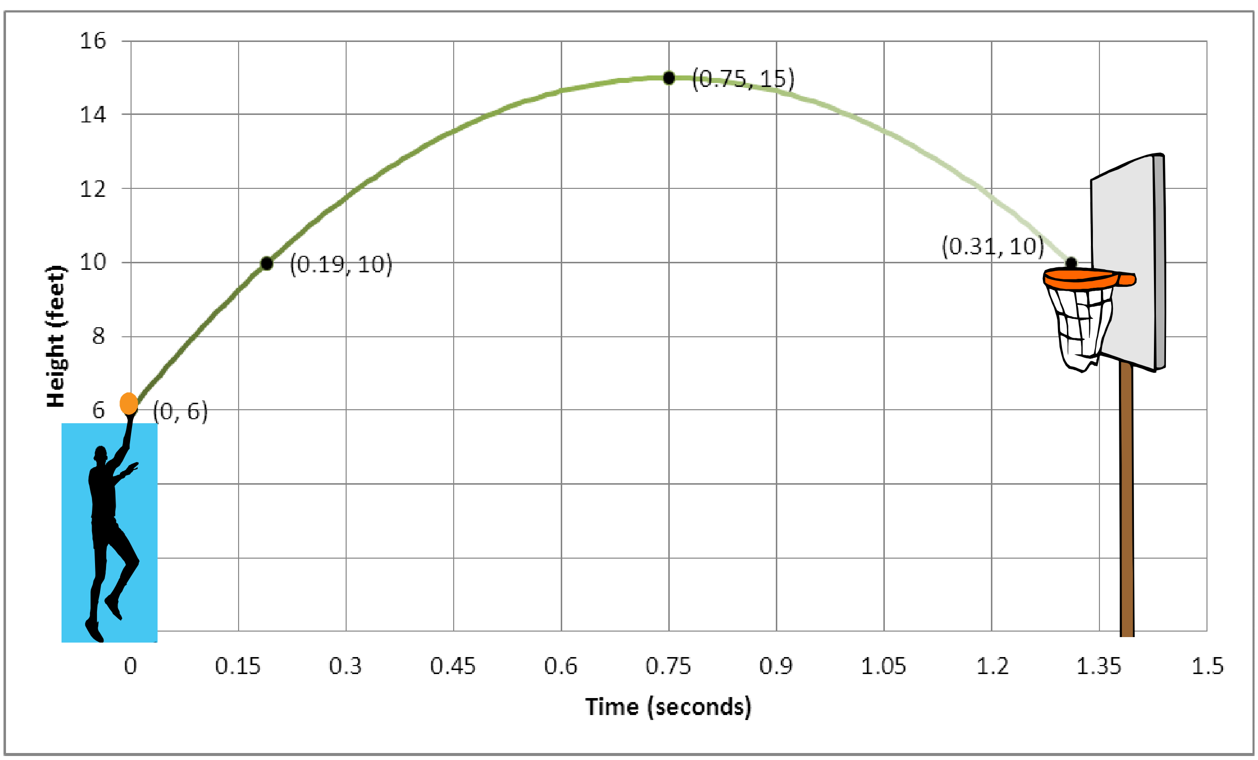
Release height = 6 ft, Initial Vertical Velocity = 20 ft/sec



Release height = 6 ft, Initial Vertical Velocity = 22 ft/sec

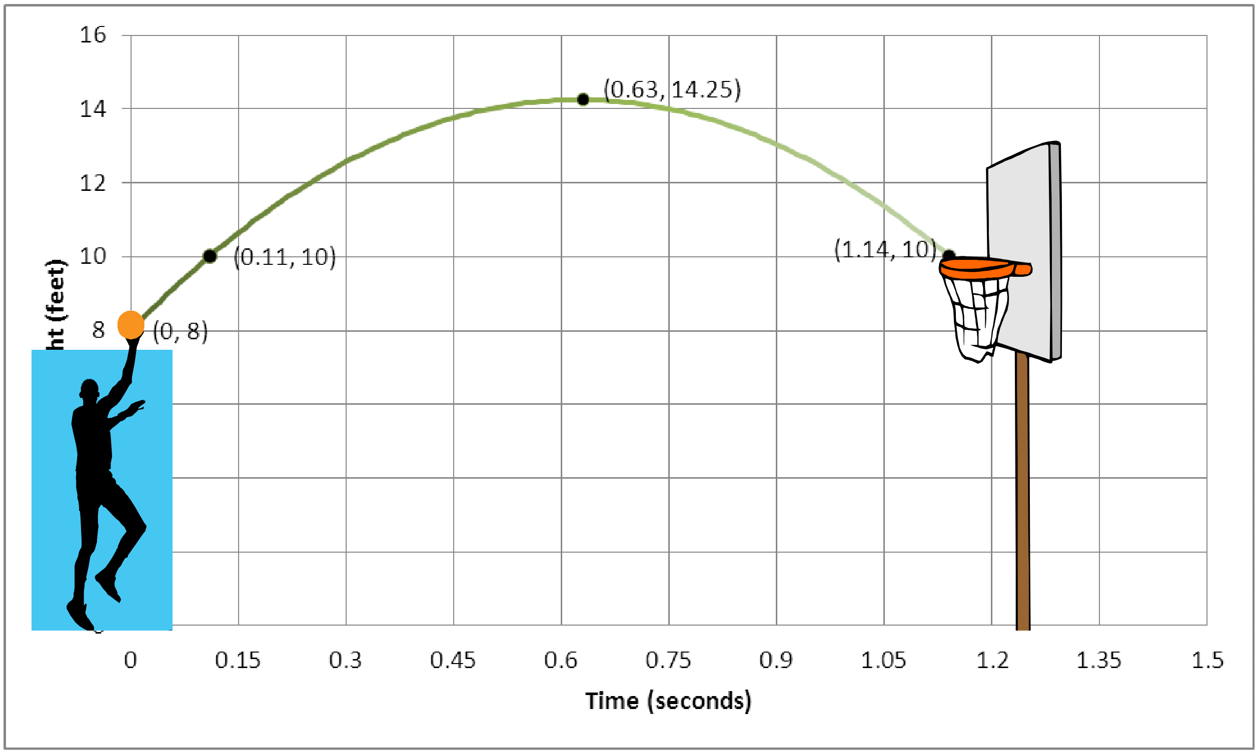


Release height = 6 ft, Initial Vertical Velocity = 24 ft/sec

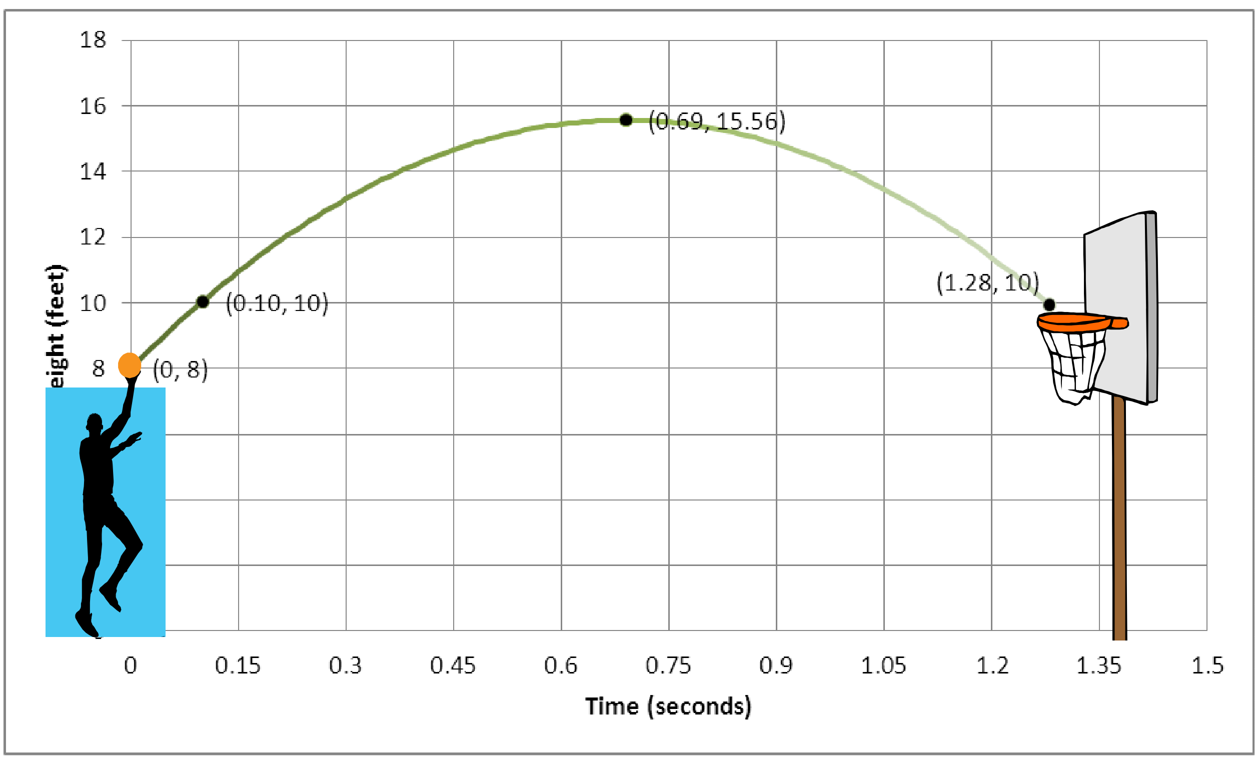


Math in Basketball: Try other challenges *Answer Key*

Release height = 8 ft, Initial Vertical Velocity = 20 ft/sec



Release height = 8 ft, Initial Vertical Velocity = 22 ft/sec



Release height = 8 ft, Initial Vertical Velocity = 24 ft/sec